Crossed Flexure Pivots Applied to a Very Broad Band Horizontal Seismometer

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Summary

Seismometers require precision suspension systems to allow them to respond to ground motions that are on the order of a few microns P-V. Significant improvements can be made to advanced amateur instruments by applying the principles of precision engineering to the design of these suspension systems. Wire flexures and crossed pivot flexures are analyzed and compared, and an example of an instrument with crossed flexure pivots is illustrated.

What is a seismometer?

Any instrument used for the detection of earth motion may be called a seismometer. This includes accelerometers, strain meters (extensometers), and inertial, or pendulum seismometers. The Manual of Seismological Observatory Practice[1] provides the following description of pendulum seismometers.

"Pendulum seismometers consist essentially of an inertial mass suspended on elastic members within a rigid framework. When the frame is disturbed by the passage of an earthquake wave, the inertia of the mass reacts against the forces transmitted through the suspension, and a relative motion occurs between the mass and the frame. These relative motions are detected and magnified by mechanical, mechanical-optical, electromagnetic or electronic methods."

The output signal from a seismometer is often proportional to the relative velocity of the elements. This means that the effects of long term drift in the suspension or tilt of the base is attenuated.

In the case of the instrument illustrated, the motion of the pendulum is controlled by a PID feedback loop that provides extension of the period of the pendulum from 12 seconds to approximately 70 seconds. The feedback loop also critically damps the pendulum. This results in a response that is flat with respect to velocity from .015 Hz to 10 Hz. The gain of the system is 90,200 Volt-seconds/ Meter.

The Lehman seismometer

The earliest description I have found of an amateur seismometer is on pages 171 and 172 of the book "Our Trembling Earth"[2]. An instrument is described that has an horizontal pendulum suspended by a wire pivot and a sharpened pin in an agate seat. The pin is actually a phonograph needle. The pendulum is damped by an eddy current damper and the position of the pendulum is sensed optically.

A similar, somewhat more modern instrument was described in the Amateur Scientist column in the July 1979 issue of Scientific American[3]. The author describes a seismometer built by James D. Lehman of James Madison University in Harrisburg, VA. This instrument is essentially identical to the earlier reference, except that it utilizes an electro-dynamic sensor to generate an electrical signal proportional to the relative velocity of the pendulum with respect to the frame. Instruments built with horizontal pendulums have become known as Lehman Seismometers and are the mainstay of amateur seismology.

The Challenge

The basic challenge is to design a high Q pendulum with a natural period greater than 10 seconds. This corresponds to a simple pendulum 25 meters long. Clearly this is too large to put in the garage. One solution is shown in figure 1. A (nearly) horizontal pendulum can be constructed that has an arbitrarily long period, although thermal effects and ground tilt limit the practical period to something less than 20 seconds. If L = 200 mm, and T = 10 seconds, α = .008 radians.

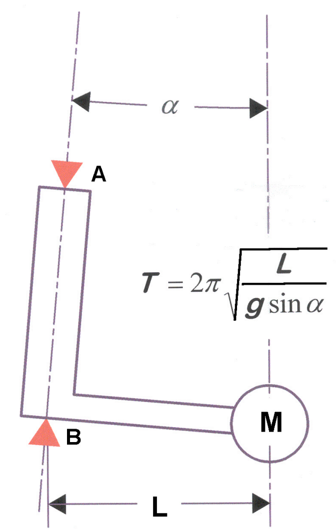

Figure 1: A horizontal "Garden Gate" pendulum

Potential solutions

The classical solution to the suspension problem is to use a wire flexure in tension at the top of the pendulum vertical arm. This wire must be angled as shown in figure 2 so that it supports the weight of the pendulum as well as the horizontal reaction caused by the overturning moment. The support at the bottom of the pendulum is classically a sharpened pin that is located in a dimple formed in the pendulum frame although occasionally one sees a hatchet blade located in a scribed groove. Another solution uses a small ball resting on a hard surface such as a carbide cutting tool insert. The ball, however, is a poor choice as it rolls on the mating surface and thus alters the line of support of the pendulum. Finally, flat flexures are sometimes used either for the upper support, or for both of the bearings.

All of these support methods fail in some manner. Most of the upper supports are too compliant along the Y direction (Perpendicular to the page) and allow the pendulum to vibrate in roll. Wire flexures used as upper supports also are too compliant in tension and lead to troublesome vibration in pitch. The lower supports are generally overstressed, even though the reaction forces are typically on the order of 20-25 Newtons.

After studying the problem, and constructing one less than satisfying instrument, it was decided to consider the use of crossed flexure pivots at both the upper and lower support points. Before implementing this approach, a comparison of crossed flexure pivots and short wire flexures was completed.

Wire flexures

The load carried by the supports is slightly less than 20 Newtons, so a value of 20 Newtons was chosen for the comparison. The wire size was selected to be a compromise between the smallest diameter that would safely carry the load and a diameter that could be handled, given reasonable care. This was deemed to be 0.30 mm. The length of the flexure was chosen to be 6 mm. This is also a compromise between minimum bending stiffness and maximum tensile stiffness. The mathematics is straightforward, with the possible exception of the calculation for centerline shift. This formula, along with others used for the crossed flexure calculation are from Stuart Smith's book entitled "Flexures, Elements of Elastic Mechanisms"[4]. The following is a summary of the results of the calculations for the selected wire flexure.

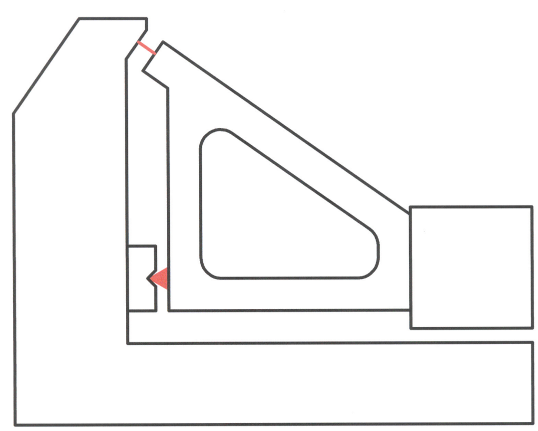

Figure 2: The classical pendulum support

Wire Flexure Math

E = 200 x 10^9 Pa, Dia = 0.300, L = 6.00

I = 3.976 x 10^{-4}

Tensile stiffness = $2\pi R^2 E/L$ = 2.356 x 10^6 N/M

Bending stiffness = EI/L = 1.33 x 10^{-2} N-M/Rad

Stress with 20 N load = 20/ πR^2 = 283 Mpa

CL shift @ 1mR = L(2Sin(/2)/ -Cos(/2)) = 7.5 x 10^{-10} M

Max stress @ 10 mR = MR/I = 50 MPa

Crossed flexure pivots

Crossed flexures have the reputation of being difficult to manufacture and align; however with care and attention during the design process, the difficulties can be minimized. The flexures used fro this example consist of small pieces of .025 mm. Stainless steel shim stock 12.7 mm wide. Two holes are punched for clamping screws and a third hole is made at the active area of the flexure. The material on one side of this hole is removed with a sharp knife leaving a passage for the second, identical flexure. Figure 3 illustrates this construction.

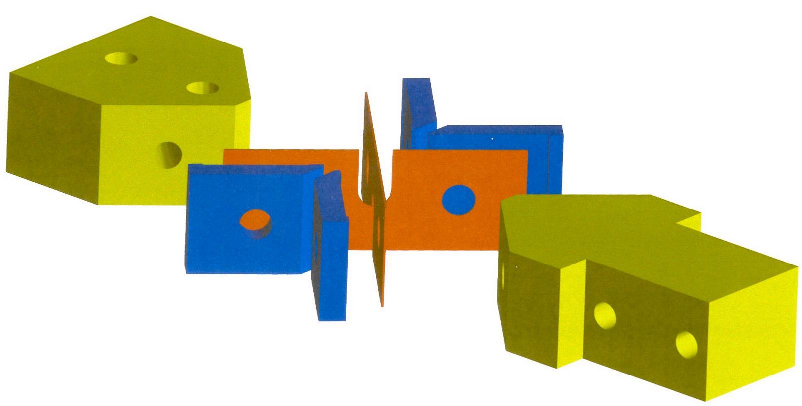

Figure 3: The construction of a crossed flexure pivot.

The dimensions chosen for the flexure in our example are: Thickness, .025 mm., Width, 5.54 mm., Length of active area, 6.00 mm. The overall width of the piece of shim is 12.7 mm. And the total length is 35.7 mm. The blocks at either end are made of aluminum, as are the four clamps. The flexures cross at 90°. As with the wire flexure, the math is straightforward.

90° Crossed Flexure Math

E= 200 x 10^9 Pa, T=.025, B=5.54, L=4.00, and

I= 7.214 x 10^{-6} mm^4.

Tensile stiffness = 2BTE/L = 14.26 x 10^6 N/M

Bending stiffness = 2EI/L = 7.21 x 10^{-4} N-M/Rad

Stress with 20 N load = 20/2cos45°BT = 102.1 Mpa

CL shift @ 1mR = 2? ^{2}L/12 = 4.71 x 10^{-10} M

Max stress @ 10 mR = MT/2I = 6.25 MPa

Comparison

Comparing the results for the two example flexures, we can see that the crossed flexures are about six times as rigid in tension as the wire and almost 20 times as compliant in bending. In addition the stress levels are substantially lower. The shift in the center of rotation is negligible for both pivots at 1 mR of deflection.1 mR is approximately the maximum operating deflection. 10 mR is the maximum deflection between hard stops. This comparison leads to the conclusion that the use of crossed flexure pivots will result in a superior suspension system for the pendulum.

Comparison

Characteristic	Wire	Flexure	Ratio
Tensile K	2.36x10^6	14.3x10^6	6.06
Bending K	1.33x10^{-2}	7.21x10^{-4}	.054
20 N Stress	283	102	.360
C/L Shift	7.50x10^{-10}	4.71x10^{-10}	.628

Application

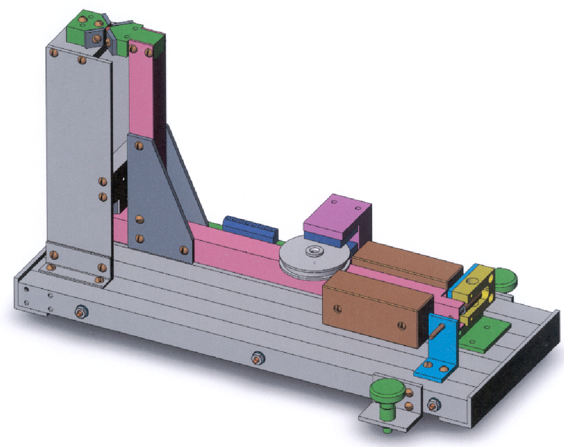

Figure 4: A horizontal pendulum seismometer with flexure pivots

The crossed flexures were incorporated into the design of a pair of horizontal pendulum seismometers that have been operating successfully for more than two years. Figure 4 is a drawing of one of these instruments. The upper flexure can be seen on the top of the column on the left. The lower flexure is located within the column and operates in tension. During the assembly of this instrument, the pivots were built up on aluminum keepers that maintained the alignment and spacing of the components. The pivots, attached to their keepers were positioned and attached to the assembly. When the assembly was complete, the keepers were removed and stored in case they should be needed again.

Figure 5 is a photograph of one of the instruments within its dust cover. A 6 inch scale is in the foreground to give scale.

Figures 6 and 7 show the upper and lower pivots. The wires near the upper pivot carry current to the motor that provides feedback that controls the motion of the pendulum.

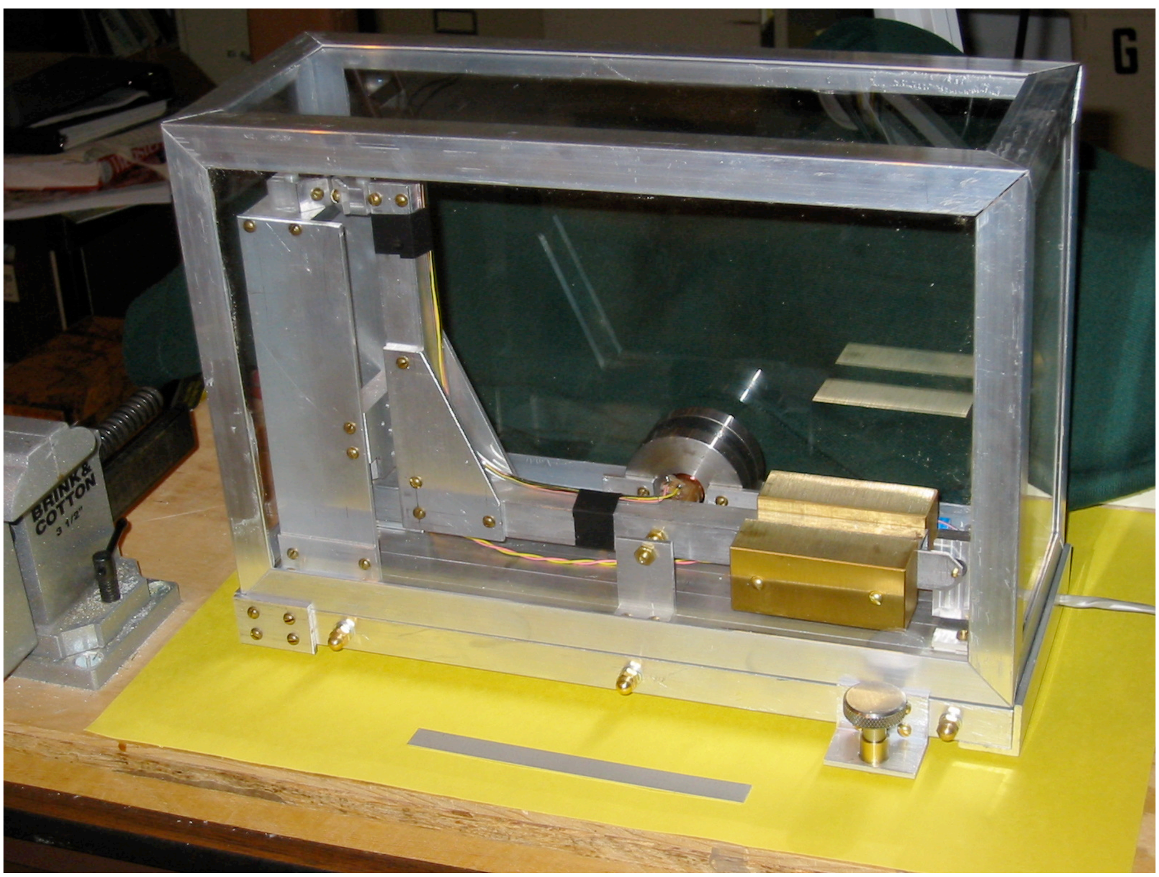

Figure 5: The completed instrument prior to installation.

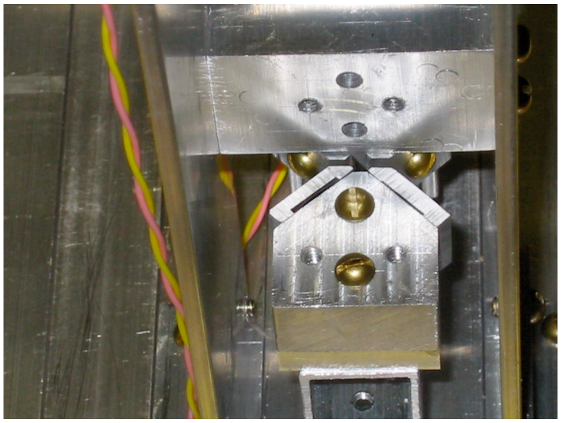

Figure 6: The lower pivot

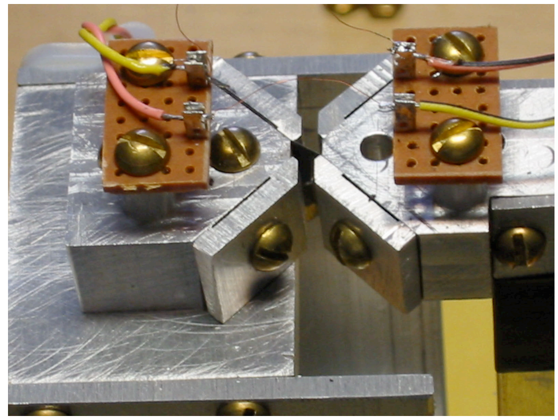

Figure 7: The upper pivot.

Conclusions

The use of crossed flexure pivots can improve the performance of many precision mechanisms. Seismometers are an example typical of many applications. For those with curiosity about amateur seismology, I have included links to the Public Seismic Network, a world-wide organization of seismologists, both amateur and professional; and IRIS, a consortium of universities for research in seismology. In addition, there are links to St Louis University where Sean-Thomas Morrissey (deceased) has documented applications of PID feedback to seismology, and the site of John C. Lahr, who is developing seismometers for use in K thru 12 educational applications.

References and links

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